CS32 Discussion
Week 7

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Outline

- Big-O Notation
- Sorting
- Tree

Complexity

- Quantify the efficiency of a program.
- The magnitude of **time** and **space** cost for an algorithm given certain size of input.
  - Time complexity: quantifies the run time.
  - Space complexity: quantifies the usage of the memory (or sometimes hard disk drives, cloud disk drives, etc.).

Size of input vs. Running time

- A program gets some kind of **input**, does something meaningful (hopefully), and produces some **output**.
- Naturally, the size of input determines how long a program runs.
  - Sorting an array of 1000 items should run a lot longer than sorting an array of 10 items.
  - But how much longer?
- Sometimes, the size of input doesn't matter.
  - Figuring out the size of a C++ string (s.size()): always the same running time.

Big Question

- Given an input of size **n**, approximately how long does the algorithm take to finish the task, in terms of **n**?

  → Big-O notation

Formal Definition of Big-O

- Let *T(n)* be the function that measures the runtime of the program given *n* size of input
- Let *g(n)* be another function defined on the real number field.
  - *T(n) = O(g(n)) iff* $\exists M > 0$ *and* $\exists N > 0$ *s.t.* $\forall n > N : T(n) \leq M * g(n)$

Upper bound

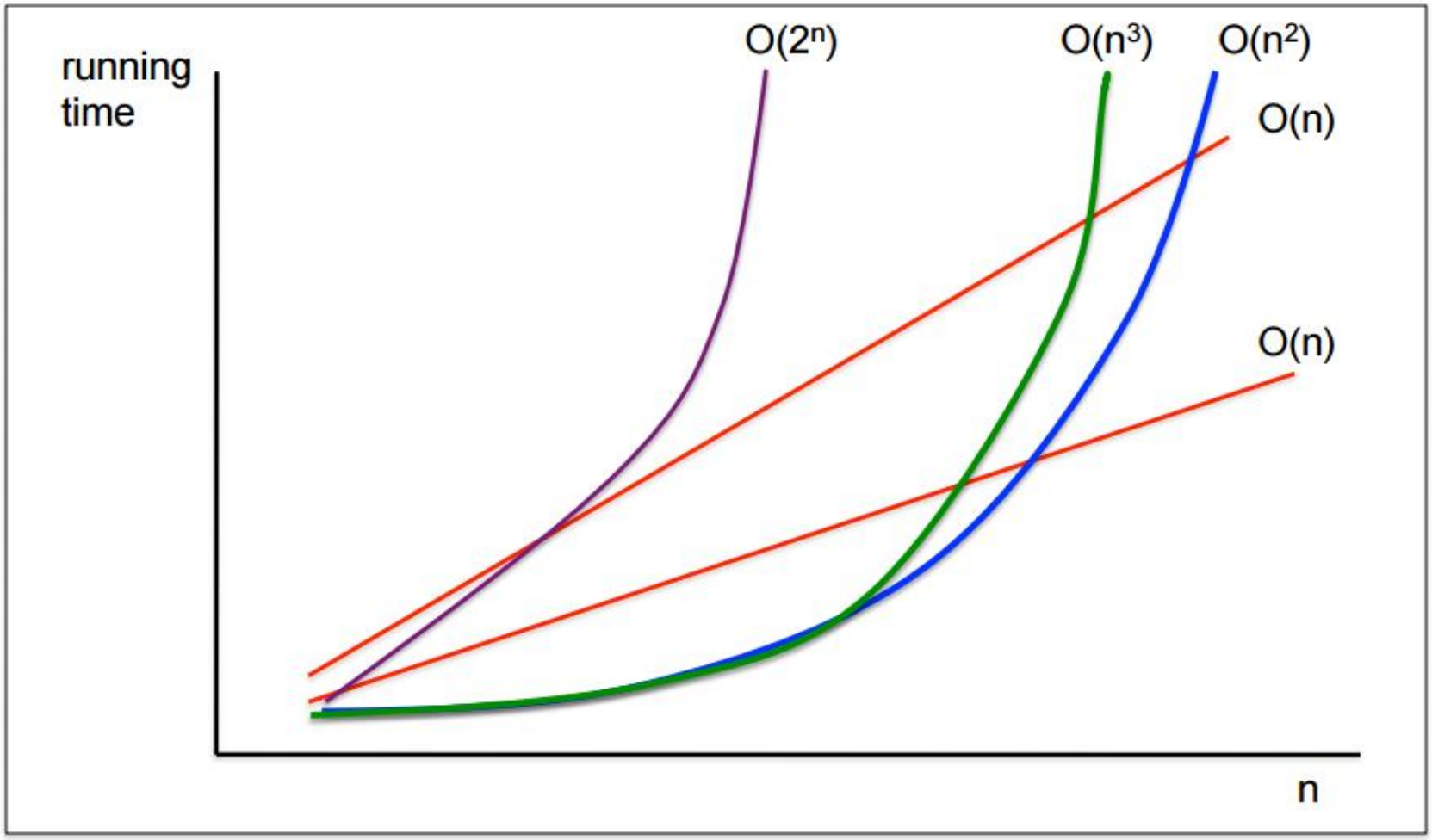
When the input size *n* grows above certain scale *N*, the runtime T(n) is of the same or less magnitude of the function *g* inside O( )

Big-O

- If your algorithm takes …
  - about n steps: O(n)
  - about 2n steps: O(n)
  - about n^2 steps: O(n^2)
  - about $3n^2 + n$ steps: O(n^2)
  - about 2^n steps: O(2^n)

- Which one grows faster in the long term?
  - 10000n vs. $0.00001n^2$

Big-O



Efficiency

- Algorithms are considered efficient if it runs reasonably fast even with a large input.
- Algorithms are considered inefficient if it runs slow even with a small input.

- For more precise definitions, take CS180 and 181. In this class, we will use these simple intuitions to analyze algorithms.

Linear Search

- Unsorted array – look for an item

```
linear_search( array arr, size n, value v )
{
   for (i=0 to n-1)
   {
      if (arr[i] == v)
         return i;
   }
   return -1;
}
```

Best case?

Average case?

Worst case?

Linear Search

- Unsorted array – look for an item

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linear_search( array arr, size n, value v )
{
   for (i=0 to n-1)
   {
      if (arr[i] == v)
         return i;
   }
   return -1;
}
```

Best case?
v is found in the first slot (a[0]) – takes 1 step

Average case?
takes n/2 = ½n steps (assuming v can appear at any location in the array with an equal probability)

Worst case?
not found – n steps

Linear Search

- Unsorted array – look for an item

```
linear_search( array arr, size n, value v )
{
   for (i=0 to n-1)
   {
      if (arr[i] == v)
         return i;
   }
   return -1;
}
```

Best case?
v is found in the first slot (a[0]) – **O(1)**

Average case?
O(n) (assuming v can appear at any location in the array with an equal probability)

Worst case?
not found – **O(n)**

All Pairs

- Find all ordered pairs

```
all_pairs( array arr, size n )
{
   for (i=0 to n-1)
      for (j=0 to n-1)
         if (i ≠ j)
            print "{arr[i] arr[j]}";
}
```

Best case?

Average case?

Worst case?

All Pairs

- Find all ordered pairs

```
all_pairs( array arr, size n )
{
   for (i=0 to n-1)
      for (j=0 to n-1)
         if (i ≠ j)
            print "{arr[i] arr[j]}";
}
```

Best case?

Average case?

Worst case?

All $O(n^2)$

Unit Operations (O(1) operations)

- Addition, Subtraction, Multiplication, Division
- Comparison, Assignment
- Input, Output of a small value (e.g. short string, an integer, etc.)

- If O(1) operations are repeatedly done in a loop for n times, then that loop is O(n).
- If this loop is within a loop that repeats n times, then this outer loop takes $O(n^2)$.

Big-O Arithmetic

- More generally:
- If things happen sequentially, we add Big-Os.
  - O(1) operation followed by O(1) = O(1) + O(1) = O(1)
- If one thing happens within another, then we multiply Big-Os.
  - O(1) operation within a O(n) loop = O(1) x O(n) = O(n)

- $O(f(n)) + O(g(n)) = O(\max(f(n), g(n)))$
- $O(f(n)) \times O(g(n)) = O(f(n) \times g(n))$

Linear Search

- Unsorted array – look for an item

```
linear_search( array arr, size n, value v )
{
    for (i=0 to n-1)                O(n)
    {
        if (arr[i] == v)            O(1)
            return i;
    }
    return -1;
}
```

Best case?
v is found in the first slot (a[0]) – **O(1)**

Average case?
O(n) (assuming v can appear at any location in the array with an equal probability)

Worst case?
not found – **O(n)**

Usually, assessing complexity involves counting nested loops – only one loop means it's likely to be **O(n)**

All Pairs

- Find all ordered pairs

```
all_pairs( array arr, size n )
{
  for (i=0 to n-1)  O(n)
    for (j=0 to n-1)   O(n)
      if (i ≠ j)                          O(1)
        print "{arr[i] arr[j]}";
}
```

Best case?

Average case?

Worst case?

All $O(n^2)$

Order of Complexity

Big O	Name	n = 128
O(1)	constant	1
O(log n)	logarithmic	7
O(n)	linear	128
O(n log n)	"n log n"	896
$O(n^2)$	quadratic	16192
$O(n^k)$, k >= 1	polynomial	
$O(2^n)$	exponential	10^{40}
O(n!)	factorial	10^{214}

Binary Search

- Find an item v in a **sorted** array

```
binary_search( array arr, value v, start index s, end index e )
{
  if (s > e)
     return -1
  find the middle point i = (s + e) / 2
  if (arr[i] == v)
    return i
  else if (arr[i] < v)
    return binary_search(arr, v, i+1, e)
  else
    return binary_search(arr, v, s, i-1)
}
```

Best case?

Average case?

Worst case?

Binary Search

- Find an item v in a **sorted** array

```
binary_search( array arr, value v, start index s, end index e )
{
   if (s > e)
      return -1
   find the middle point i = (s + e) / 2
   if (arr[i] == v)
      return i
   else if (arr[i] < v)
      return binary_search(arr, v, i+1, e)
   else
      return binary_search(arr, v, s, i-1)
}
```

Best case?
O(1) – by now you can see the best case analysis doesn't help much

Average case?
O(log n)

Worst case?
O(log n)

Binary Search

- At every iteration, we divide the search space in half.
- You keep dividing the size by 2 until it becomes 1.
- It takes ~ $\log_2 n$ steps to get to 1.
- $\log_{10} n = (1 / \log_2 10) \log_2 n$
- So the base does not matter.

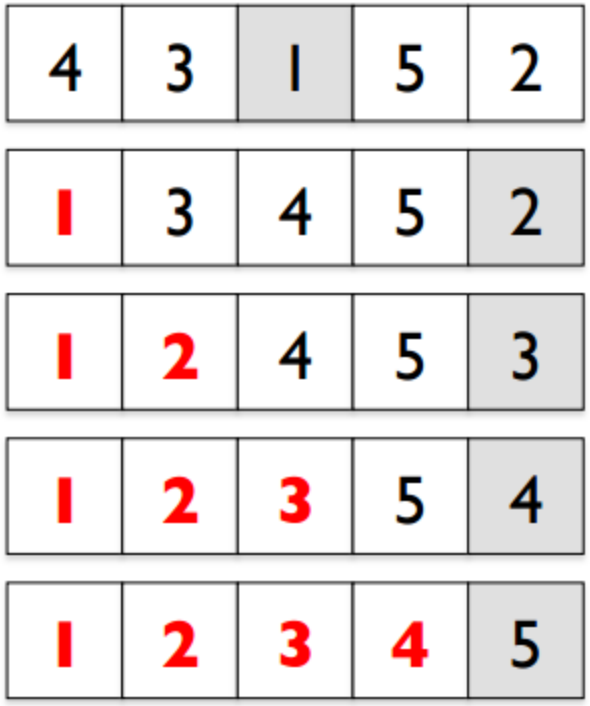
Why Big-O's are important

- You'll be asked about it in job interviews!!!!!

Sorting Algorithms

- We now switch gears and discuss some well known sorting algorithms.

Selection Sort



- Find the smallest item in the unsorted portion, and place it in front.
- What is the running time (complexity) of this algorithm?

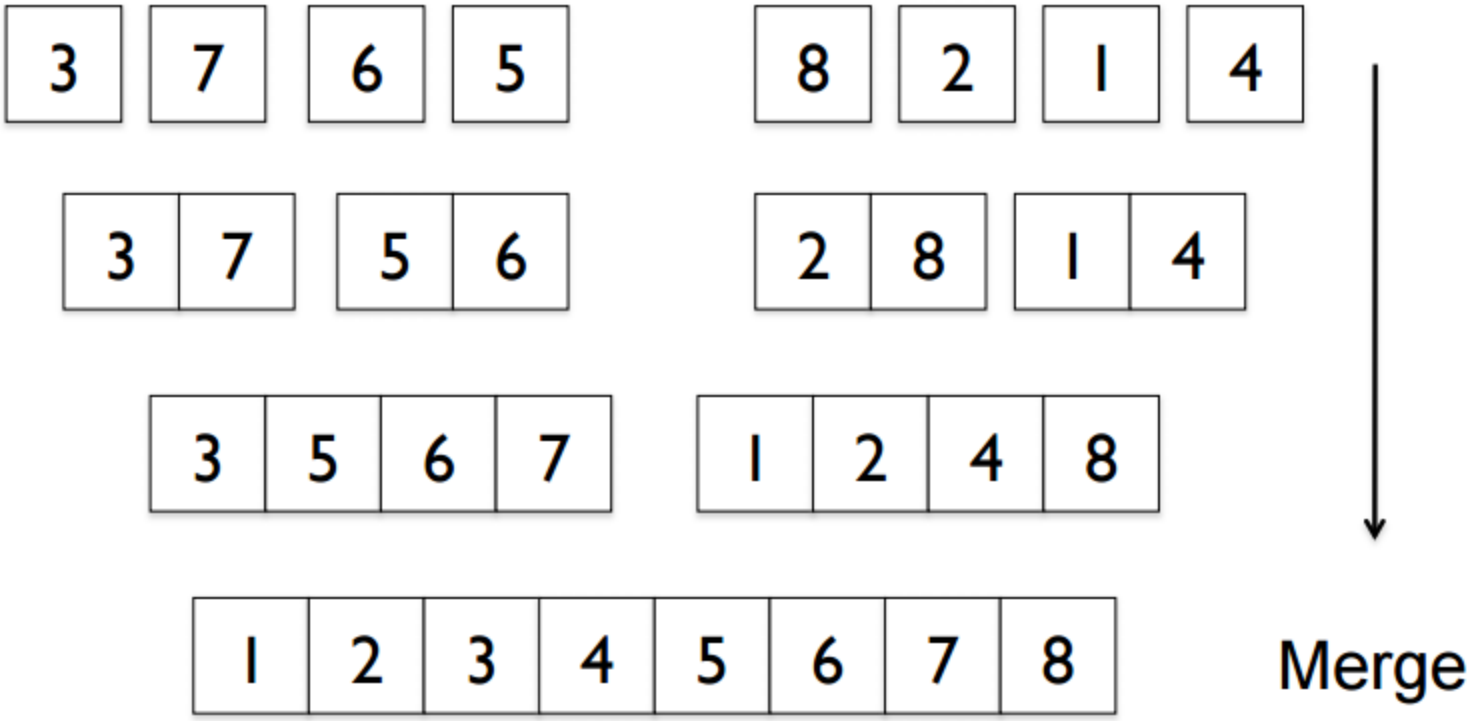
Insertion Sort

- Pick one from the unsorted part, and place it in the "right" position in the sorted part.
- Best case?
- Avg. case?
- Worst case?

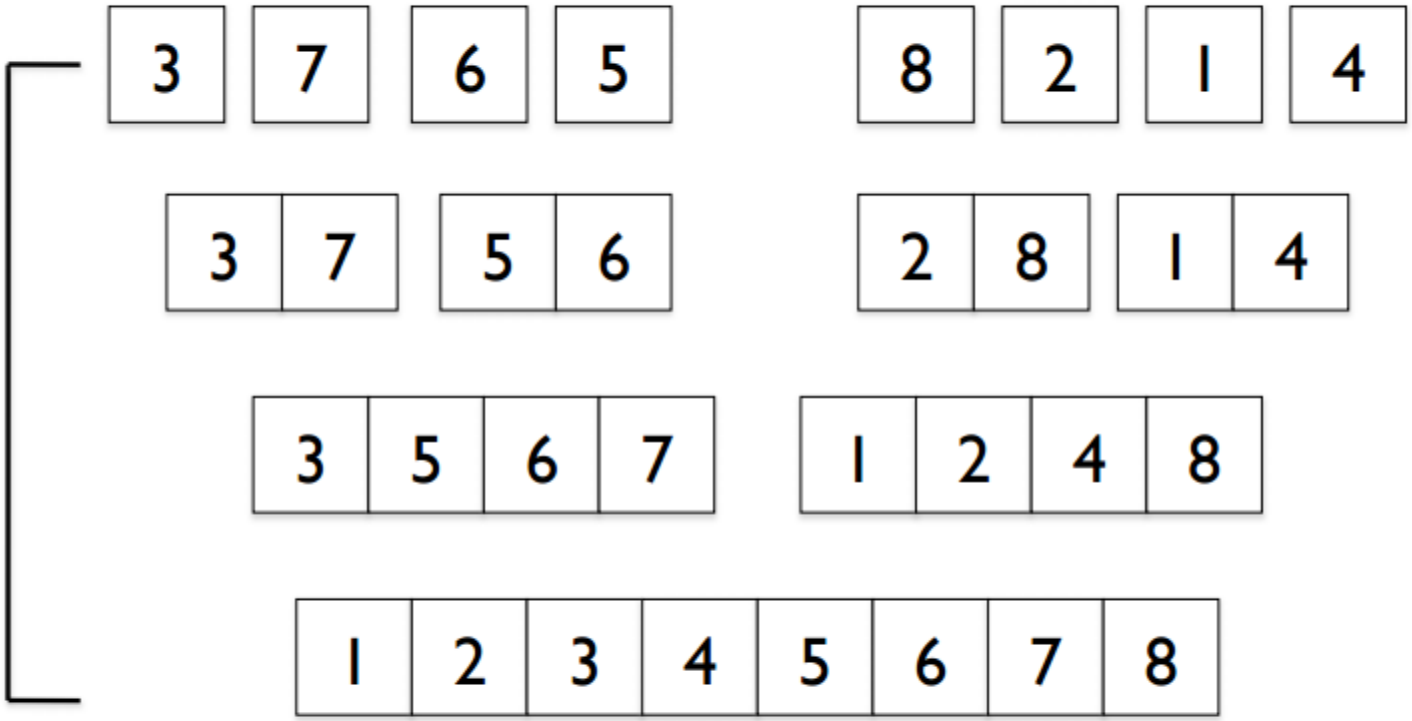
Insertion Sort

- Pick one from the unsorted part, and place it in the "right" position in the sorted part.
- Best case? **O(n)**
- Avg. case? **$O(n^2)$**
- Worst case? **$O(n^2)$**

Merge Sort



Merge Sort: Running Time?



$O(n)O(\log n) = \mathbf{O(n \log n)}$

General Sorting: Running Time

- O(n log n) is faster than $O(n^2)$ – merge sort is more efficient than selection sort or insertion sort.
- O(n log n) is the best average complexity that a general (comparison) sorting algorithm can get (assuming you know nothing about the data set).
- With more information about the data set provided, you can sometimes sort things almost linearly.

Quick Sort

- Pick a **pivot**, and move numbers that are less than the pivot to front, and ones that are greater than the pivot to end. (Does this sound familiar?)
- On average, O(n log n)
- Depending on how you pick your pivots, it can be as bad as $O(n^2)$

Permutation

```
void permutation(vector<int>& nums, int start) {
      if (start == nums.size() - 1) {
           for(int i=0; i<nums.size(); ++i)
               cout << nums[i] << ' ,';    cout << endl;
      }
      permutation(nums, start + 1);
      for (int i=start+1; i<nums.size(); ++i) {
         swap(nums[start], nums[i]);
         permutation(nums, start + 1);
         swap(nums[start], nums[i]);
      }
   }
permutation(nums, 0); //call this function
```

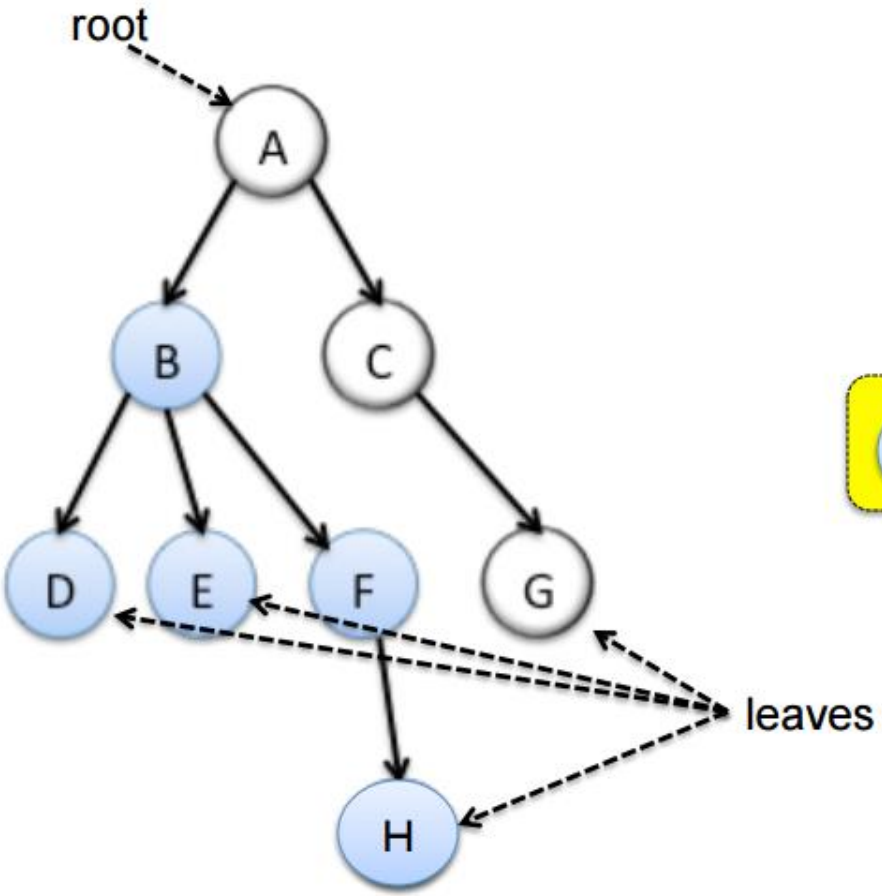
O(n!)

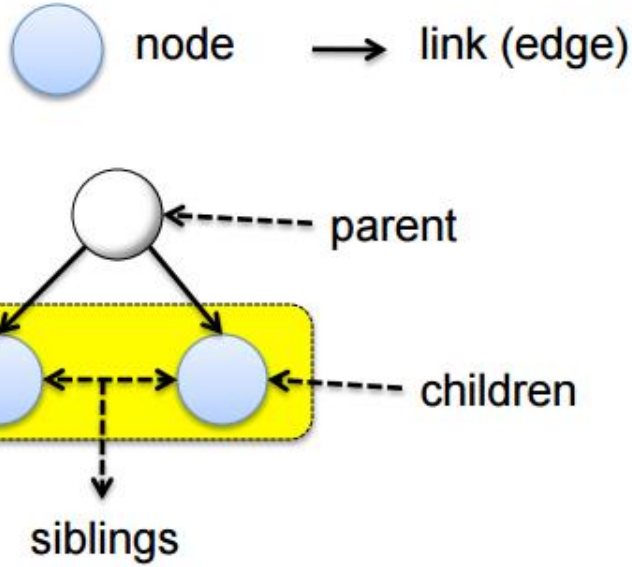
Quick Questions

- Given an unsorted array of n items, what is the best you can do to search for an item, if you are to run this search only once?
- Given an unsorted array of n items, what is the best you can do to search for an item, if you are to run this search 100 times? (assume: n >> 100)
- Given an unsorted array of n items, what is the best you can do to search for an item, if you are to run this search n times?

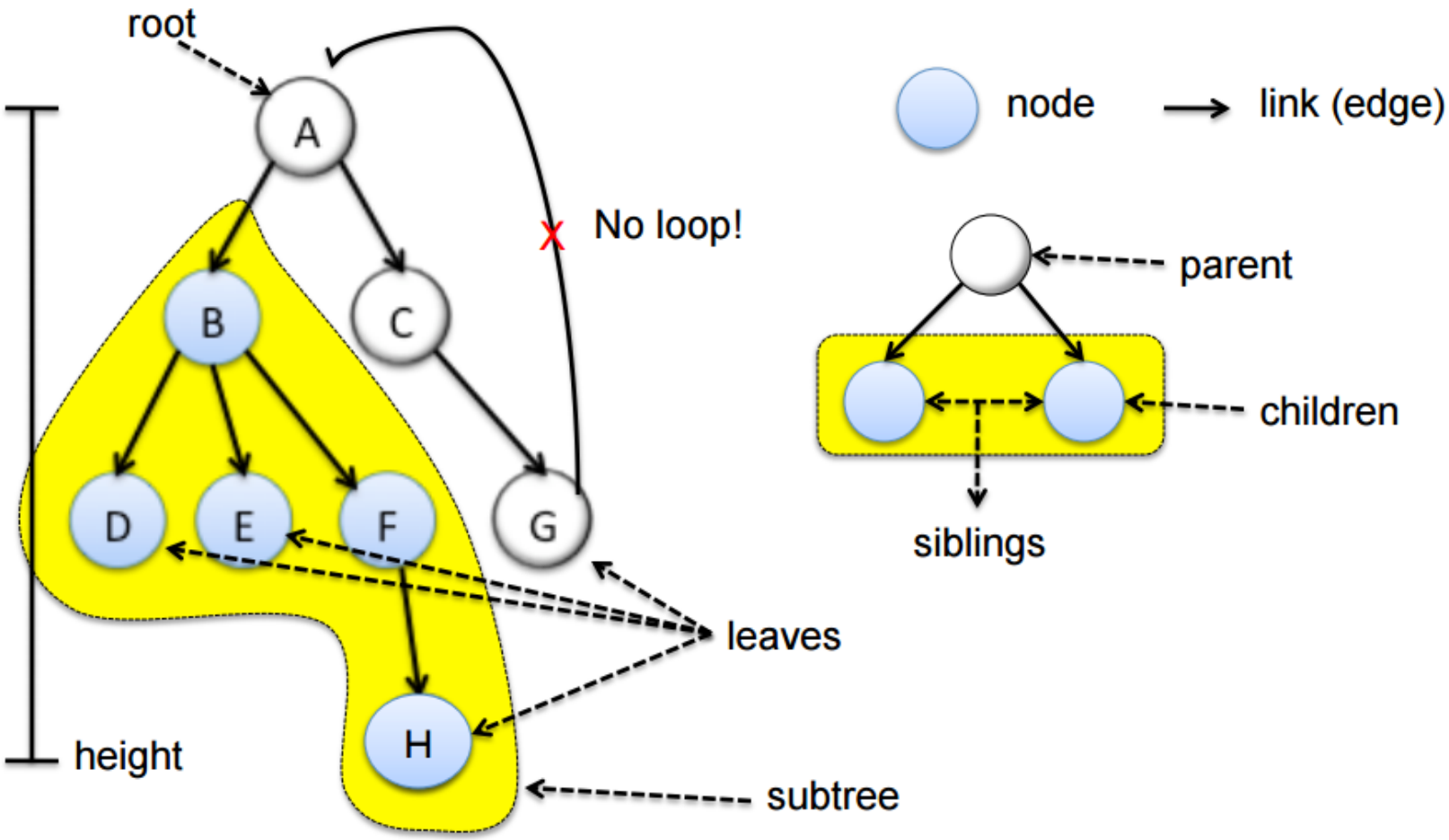
Tree

Tree: Definitions

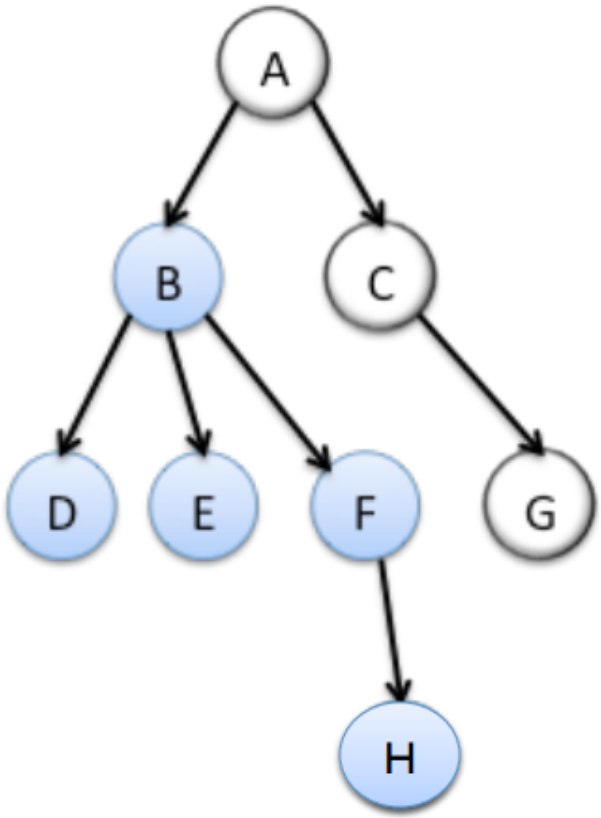




Tree: Definitions



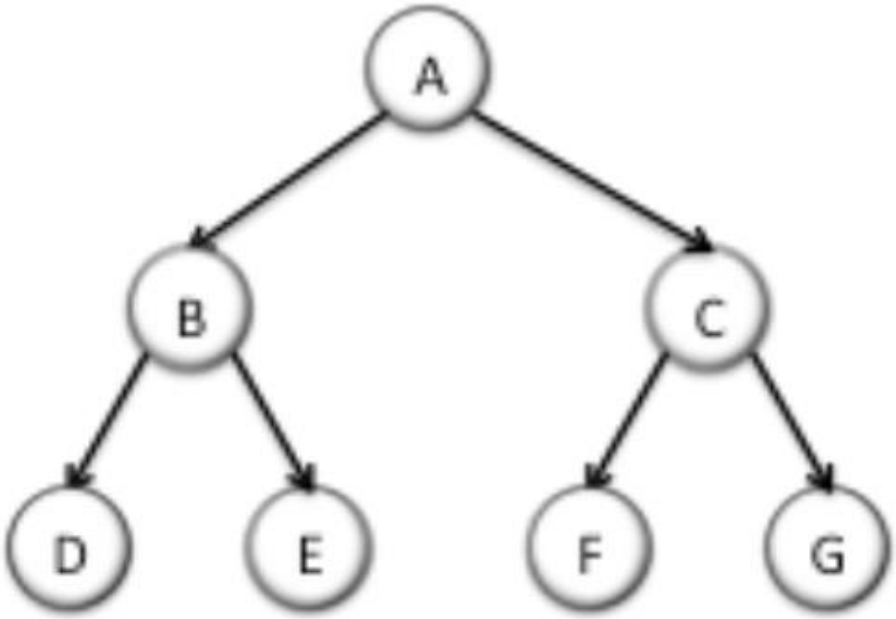
Bound on # of edges



How many edges should there be in a tree of **n** nodes?

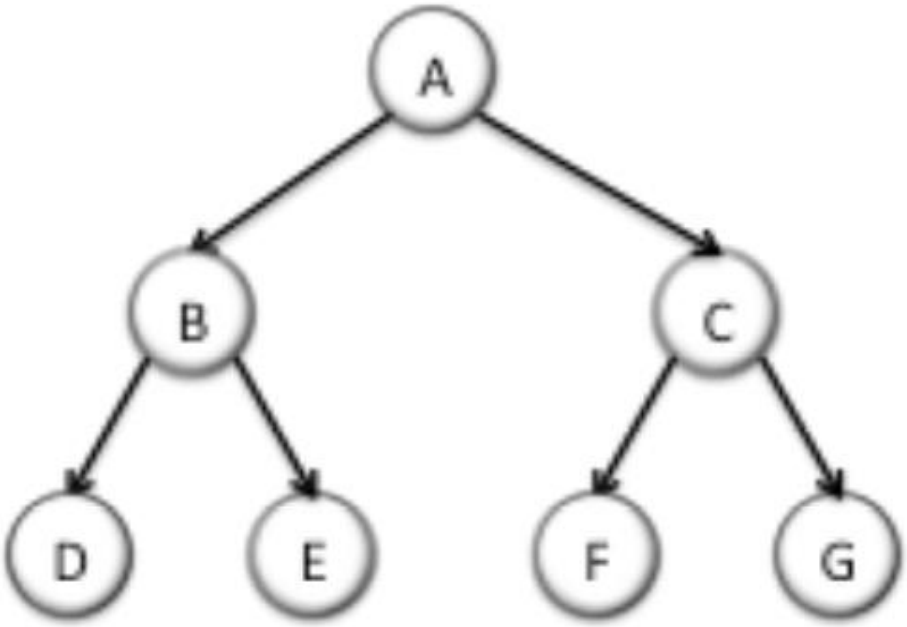
n - 1

Binary Trees



No node has more than 2 children (left child + right child).

Binary Trees



$2^0+2^1+\ldots+2^h=2^{h+1}-1$

How many nodes can a binary tree of height ***h*** have?
(one with max. # of nodes == full binary tree)

Tree is a data structure!

- For every data structure we need to know:
  - how to insert a node,
  - how to remove a node,
  - search for a node

- and (for tree only)
  - how to traverse the tree

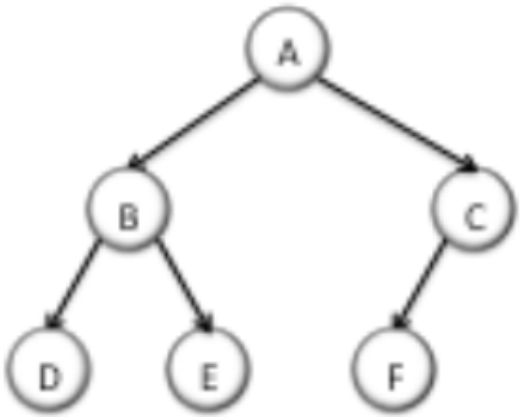
```
struct Node
{
    ItemType val;
    Node* left;
    Node* right;
};
```

Three Methods of Traversal

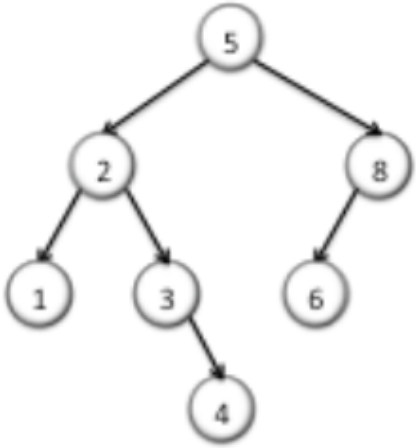
```
void preorder(const Node *node)
{
  if (node == NULL) return;
  cout << node->val << " ";
  preorder(node->left);
  preorder(node->right);
}
```

```
void inorder(const Node *node)
{
  if (node == NULL) return;
  inorder(node->left);
  cout << node->val << " ";
  inorder(node->right);
}
```

```
void postorder(const Node *node)
{
  if (node == NULL) return;
  postorder(node->left);
  postorder(node->right);
  cout << node->val << " ";
}
```



treeHeight




```
int treeHeight(const Node *node)
{
    if (node == NULL)
        return 0;

    int leftHeight = treeHeight(node->left);
    int rightHeight = treeHeight(node->right);

    if (leftHeight > rightHeight)
        return leftHeight + 1;
    else
        return rightHeight + 1;
}
```

Bugs in your software are actually special features :)